Modern Mathematics for the Older Slower Learner

J. J. Marshall

"Mathematics is a difficult subject, but not as difficult as it is often made out to be. Most people have a greater capacity for mathematical understanding than they are aware of, and a large reservoir of undeveloped mathematical competence certainly exists among youngsters of ordinary ability which good teaching and an enlightened approach could reveal. Few, if any, of our pupils are ever likely to become mathematicians; but some may well come to find satisfaction in mathematical work if its purpose has first been clearly seen and confidence established through the successful use of mathematics as a tool" (Newsom Report 1963).

The Newsom Report goes on to say that "even the New Mathematics way have something to offer for some of our pupils where there is a well-qualified and well-versed enthusiast on the staff". Does this then imply there are types of mathematics ? It reminds me of a student who came to see me after visiting her secondary school to arrange next year's work with her Head of Department, to tell me that she wouldn't be able to do any modern mathematics next year as "we are doing Nuffield Mathematics from September".

The transformation is on. The Sunday papers tell us that the New Mathematics may well outdo the initial Teaching Alphabet as the "fastest growing education reform" (Chapman 1967). There are three main projects well under way*, teachers' courses abound and books are appearing daily, yet somehow there has grown this dichotomy and it appears that there are types of mathematics.

The marriage of education and mathematics

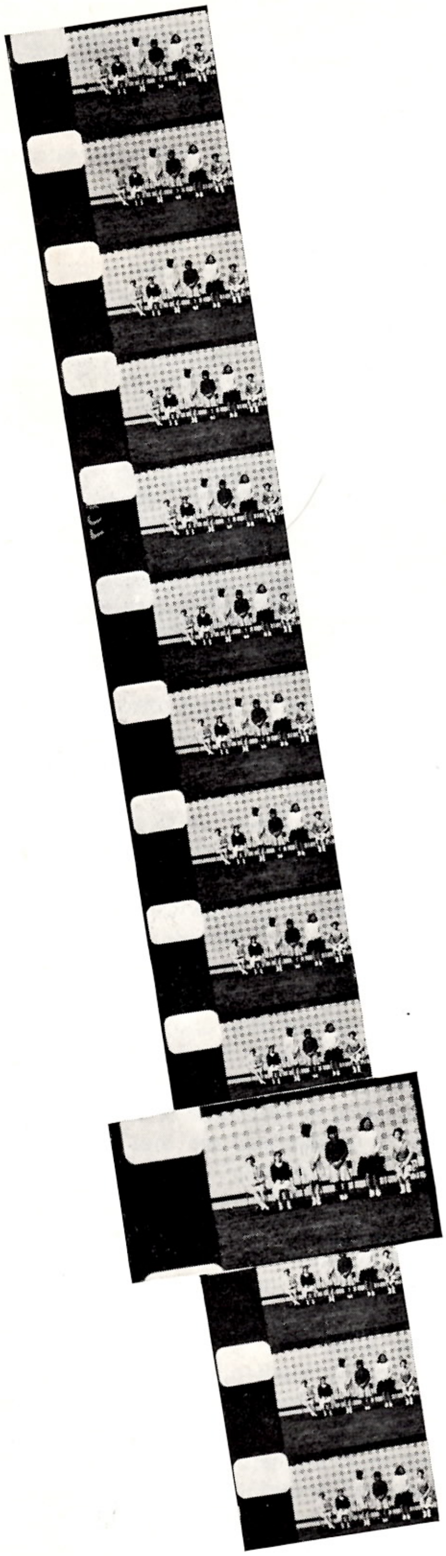
If this is so then perhaps the editor has invited the wrong person to write this article. What has turned out to be *Modern Mathematics* was for me mathematics taught in what I felt was a more enlightened way. It was a marriage of education and mathematics: and as such, satisfied a definition of mathematics as "the discipline for handling pure abstractions" (Battersby 1966) and a desire that the syllabus should "foster in them a critical, logical, but also creative, turn of mind" (Nuffield Foundation 1964). With the sort of child we meet in remedial education, I was seeking an attitude which would cause a child to say in later years, "Mathematics, yes I enjoyed it, I thought I understood it. I wasn't always very good at it but I saw the need for it". Instead of saying "Mathematics—ugh !"

The Initial Teaching Alphabet is not an alphabet to replace our existing twenty-six letters but rather an aid by which young children can come to gain a greater facility with the everyday situation. Modern mathematics to me is its counterpart. In 1957 the syllabus I was given included "multiplication of decimals". I had the task of explaining that $0{\cdot}1 \times 0{\cdot}1 = 0{\cdot}01$. My colleagues had two views on the matter. One was simply to tell them to multiply as normal and then count up the number of decimal places ---; the other, more enlightened, was all for explaining that the answer must be 0·01 as $1/10 \times 1/10 = 1/100$---. I disliked the first as it didn't really seem educational, while to most of the children I was dealing with 1/10 of 1/10 seemed frightening and outside their experience. Could they in fact appreciate 1/10 ?

Structure and operations

Modern mathematics, then, was born out of a desire to put over ideas which had not been grasped, by the more traditional methods of the time. A scale such as the binary scale would give things like $0{\cdot}1 \times 0{\cdot}1 = 0{\cdot}01$ which now means $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. This I found more acceptable as $\frac{1}{2}$ of $\frac{1}{2}$ was more within the child's experience. The problem now was to introduce

*M.M.E.—The Midlands Mathematics Experiment.
The Nuffield Foundation Mathematics Teaching Project.
S.M.P.—The School Mathematics Project.



the Binary Scale in such a way that the child could understand both structure and operations. It was no use putting the Binary Scale on the board saying "Learn that, there will be a test tomorrow". In order to stimulate an involvement in the Binary Scale a "Human Computer" was devised (Marshall 1963). This consisted of a row of children sitting on chairs at the front of the class. Each person was told she could do two things: stand up and sit down. Hence the two state system required. The computer becomes automatic in transmission if the children are told to move when the person on their left sits down (if they are sitting they stand, if they are standing they sit). When a person is standing up she represents one and when she is sitting down a zero. Thus the number 14 is represented by the 2nd, 3rd and 4th person from the right (their left) standing with the 1st sitting down, i.e. 1110. The 'machine' is set in motion by telling the first person (i.e. the girl on the right) to stand up and sit down repeatedly. When she stands up 1 is registered. When she sits down the next person stands up representing 2 (10), while for 3 the first person stands again (11). All the movements come initially from the first person. When she sits down again (she has moved four times) the next person moves, that is, she sits. As she sits the third person stands up to represent 4 (100). And so it goes on. In this way the class can see that the first person moves 4 times before the 3rd person stands up: she moves 32 times before the 6th person stands. Eventually place value emerges as . . . 64, 32, 16, 8, 4, 2, 1. ("Sir, they are doubling as they go up".)

THE BINARY SCALE

0, 0	8, 1000	16, 10000	24, 11000
1, 1	9, 1001	17, 10001	25, 11001
2, 10	10, 1010	18, 10010	26, 11010
3, 11	11, 1011	19, 10011	27, 11011
4, 100	12, 1100	20, 10100	28, 11100
5, 101	13, 1101	21, 10101	29, 11101
6, 110	14, 1110	22, 10110	30, 11110
7, 111	15, 1111	23, 10111	31, 11111

It may seem to the reader that there is a lot of work to get through just to establish the idea of multiplication of decimals. However, the by-products involved more than justify the effect. The exciting word computer can be brought into the classroom with a look at punched tape and cards (there are some excellent films on computers which can supply background). Other number scales can be introduced (a link with Dienes) and a study of their structure made. Above all regained confidence through involvement and an opportunity **to discover.**

Discovery will mainly come with the right prompting by teacher, e.g. "Tell me something about the Binary Scale ("they are all 0's or 1's sir"). "What do you notice about the right hand column ?" ("they go 010101, sir", and even "the even numbers end in 0 and the odd numbers end in 1"). Perhaps the most surprising remark came when a boy discovered that "the numbers from 0–7 appear again from 8–15 only there is a 1 in front of them". Written work, too, must be reframed. A typical written exercise would be:

Ex. 1. Write down the following denary numbers as binary numbers:

1a 3	f 10	k 24	p 56
b 5	g 12	l 28	q 72
c 6	h 14	m 36	r 80
d 7	i 18	n 40	s 112
e 9	j 20	o 48	t 144

I would prefer to see the exercise reorganised to:

Ex. 1. Write down the following denary numbers as binary numbers:

1a 3	f 5	k 7	p 9
b 6	g 10	l 14	q 18
c 12	h 20	m 28	r 36
d 24	i 40	n 56	s 72
e 48	j 80	o 112	t 144

u. What happens to the binary number if the denary number corresponding to it is doubled ? (Hamill and Marshall 1967.)

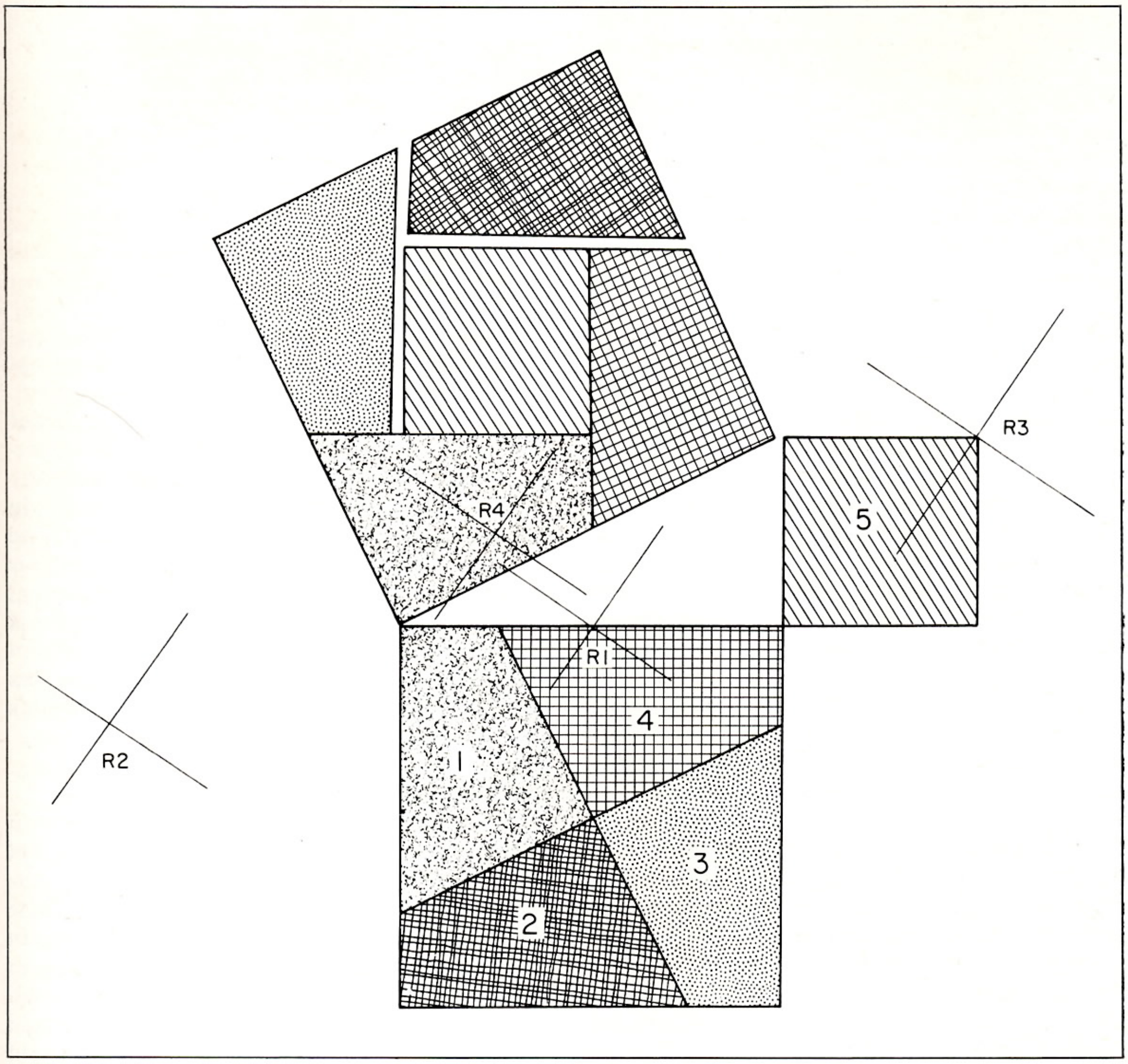
Modern mathematics to me would require the second type of exercise where possible, for the first type may be charged with just keeping the class occupied.

It was by discussion, in the third year, that place values were "extended" below. 1. We had been told earlier that 64, 32, 16, 8, 4, 2, 1 indicated a doubling up and indeed went on for ever–coming down then meant halving–and could this go on for ever ? The place values are now extended 16, 8, 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ ("Sir, you have got the same numbers underneath as you had before !") A point can separate the whole numbers from the fractions, so that 1·1 would be $1\frac{1}{2}$. This fact alone led on to other interesting aspects as we needed a symbolism to signify which scale we were working in. The binary scale 1·1 was indicated as $1{\cdot}1_{\text{two}}$. I didn't want to adopt $1{\cdot}1_2$ as with the poor co-ordination of a lot of these children the number may become 1·12. This was to fall down later on as one or two couldn't read things like $1{\cdot}1_{\text{eight}}$ when looking at other scales so for them it became $1{\cdot}1_{\text{eight}}$ with the 8 in colour (sometimes this was 'forgotten' and eventually 8 was read).

Further discussion led us to see that in $1{\cdot}1_{\text{two}}$ the 'two' tells us what base we are working in; that we had 3 halves: and that 11 in binary form was 3 (note: Can you see the 3 in 110_{two} ?) We also found out that in $\mathbf{1\frac{1}{2}}$ the 2 told us what quantities we were dealing with while the other ones corresponded fairly well with our binary scale study. We also realised that many three bar electric fires have two switches and four heats. ("Does that mean a seven bar fire would have three switches ?") By now I felt we were becoming abstract with questions, prefixed by "What happens if . . .", being provoked both by teacher and taught. "But what use is it" people would ask and "what about the mathematics they need". What about money problems; Pythagoras and Simultaneous Equations ? (!) As for 'use' and 'need', initially the problem was one of recreating interest so there was a 'need' and a 'use'. Money problems were considered ("Sir, a shillings and pence sum is a base 12 sum, isn't it ?") and often involved a ready reckoner which mainly took a binary form as 15/- could be regarded as 8/- + 4/- + 2/- + 1/-; Pythagoras was answered on a "What happens if" basis when talking about moving shapes (see page 151) and Simultaneous Equations–well, we didn't attempt that.

There is, I feel, a danger in giving the slow child a 'watered down' version of the G.C.E. syllabus and this may still hold with 'Modern Courses'. I would like to think that with more attention to teaching methods courses could be designed that would provide a 'Concentrating up': a course aimed at providing experiences which would lead to mathematical abstractions. How far along the road and how much time is devoted to each stage would depend upon the skill of the teacher, who best knows the abilities of his charges. A colleague of mine teaching in a Selective school told me that 15 minutes of the 'Human Computer' was enough, whereas with some slow children I felt they would have gladly continued all day (one really bright boy spent two minutes with the 'Human Computer' and went on to circuits, programming and logic. His "what happens if" became "I suppose you could").

Has modern mathematics then anything to offer the slow child ? I believe it has for if modern mathematics is concerned with the teaching of a mathematics not divorced from the needs and development of the student, then it is in the realm of the slow child that new thinking must occur. In the space of one article such as this, a comprehensive course for the



slow learners could hardly be outlined. What is more, I am not sure if I could outline one, for the attitudes are, I feel, still evolving.

References

BATTERSBY, A. (1966). Mathematics in Management, London, Penguin.

CHAPMAN, C. (1967). *Sunday Times*.

HAMILL, C., MARSHALL J. J. (1967). Modern Mathematics, London, Hultons.

NEWSOM REPORT (1963). Half our Future. H.M.S.O.

Bulletin No. 1 (1964).
Nuffield Foundation Mathematics Teaching Project,

MARSHALL, J. J. (1963). *The Teacher,* 2nd August.